Comments on "Vibrational Characteristics of Thin-Wall Conical Frustum Shells"

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IN a recent note, Watkins and Clary[1] reported some interesting experimental results on resonant frequencies and nodal patterns of vibrations of truncated conical shells. It was reported that for free-free conical shells, there are a greater number of circumferential nodes (of transverse displacement w) at the major edge than at the minor edge when the shell is in resonant vibration. In Fig. 2 of their note,[1] a sketch of a typical nodal pattern was shown which included two U-shaped nodal lines with both ends at the major edge. The authors remarked that special analytic procedures are needed to account for the difference in the number of circumferential nodes at the two edges.

It is felt that this phenomenon may be explained by the fact that the authors used two electromagnetic shakers to excite the resonant vibrations, with one shaker mechanically attached to each end of a generator (Fig. 2, Ref. 1). Therefore, the reported modes are steady-state forcing vibrations in resonance with some natural frequencies that may, but do not necessarily, simulate the associated normal modes of free vibrations of the tested shell. Since material damping and acoustic damping are always present in vibration experiments, the use of some kind of shaker is essential to keep the vibration continuing for measurement. However, if the purpose of the experiment is to investigate the natural frequencies and associated normal modes of the tested model, the placement of the shaker, or shakers, must be carefully designed so as to effect energy transfer with minimum distortion of the normal mode. One way to check the distortion of the modal shape is by cutting off the energy source of the shakers and examining whether there is any appreciable change of the nodal pattern during the decaying free vibration.

Since the term normal mode denotes nothing but the harmonic component resulting from mathematical decomposition of the homogeneous solution of a linear dynamic system, the nodal pattern of a normal mode of a shell of revolution (with axisymmetric boundary conditions) naturally consists of parallel circles and equispaced meridians, because the axial symmetry of the shell and circumferential periodicity of the vibration motion permit the solution of the problem by separation of variables.[2–5] A recent experimental program on vibrations of conical shells conducted at Southwest Research Institute confirms the foregoing discussion. In these experiments steel shells were excited through harmonically varying magnetic fields located near the opposite ends of a diameter of the major base, and no U-shaped nodal lines are observed in resonant vibrations of free-free or freely supported conical shells.

It might be remarked that there is another important factor in getting an "irregular" nodal pattern, namely, the dynamic system must be such that some distinct normal modes have equal or nearly equal frequencies (which is highly probable in vibrations of plates and shells); then their linear combinations will result in nodal patterns of complex nature. The nodal patterns of a vibrating square plate with free edges furnish a well-known example.

References

[1] Watkins, J. D. and Clary, R. R., "Vibrational characteristics of thin-wall conical frustum shells," AIAA J. **2**, 1815–1816 (1964); also available in more detailed form as AIAA Preprint 64-78 (January 1964).

[2] Goldberg, J. E., Bogdanoff, J. L., and Alspaugh, D. W., "On the calculation of the modes and frequencies of vibration of pressurized conical shells," *AIAA 5th Annual Structural and Materials Conference* (AIAA, New York, 1964), pp. 243–249.

[3] Kalnins, A., "Analysis of shells of revolution subjected to symmetrical and nonsymmetrical loads," J. Appl. Mech. **31**, 467–476 (1964).

[4] Kalnins, A., "Free vibration of rotationally symmetric shells," J. Acous. Soc. Am. **36**, 1355–1365 (1964).

[5] Hu, W. C. L., "Free vibrations of conical shells," NASA TN D-2666 (1956).

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Reply by Authors to W. C. L. Hu

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THE authors wish to thank Hu for his comments in regard to the conical frustum nodal patterns reported in their recent technical note.

It appears that the issues under discussion are the following: 1) Do the true nodal patterns associated with the normal modes of an idealized isotropic, axisymmetric conical frustum shell always consist of parallel circles and equispaced meridians? 2) If so, are the departures from the idealized case (due to damping, anisotropy, nonlinearities, shaker interferences, boundary restrictions, etc.) inherent in the conical frustum structures tested such that the node lines deviate from the idealized case as indicated in the authors' paper? The authors are not sure of the answer to the first question. Hu and others have suggested that the answer is yes. However, the authors doubt that the idealized mathematical analog is adequate for the general treatment of the physical structures under discussion. If the answer to the first question, a mathematical one, is yes, then the authors believe the answer to the second question, a physical one, is also in the affirmative because they are convinced that the results reported are reliable.

The authors were the first to be surprised at the results of the investigation, and many repetitions of the experiment were necessary to convince them and their associates that the results achieved were valid.

It is the authors' opinion that any influence of shaker position or of the type of shaker used was not sufficient to cause the conical frustum shells to respond in the manner reported. Comparable nodal patterns were obtained when electromagnetic shakers were placed at the following locations: a single shaker at either a major or minor diameter, or at any position along a generator; a shaker attached to each end of either a major or minor diameter; and three shakers spaced evenly along a generator. Effects of mechanical attachment to the shell can be essentially eliminated since similar nodal patterns were obtained when an air shaker[1] was used to excite the shell.

Only for limited cases involving lower-order modes ($n = 3,4,6$) were the authors able to obtain the mode shapes typical of classical, undamped, linear theory as discussed, for example, by Goldberg.[2] For the higher values of conicity (above 7.4°), classical type modes could be excited only when $n = 3$. In other studies with plates and cylinders, whenever mixed modal response was detected, further application of careful experimental procedures, such as followed in this investigation and in Ref. 3, resulted in exciting the uncoupled modes for the system. However, contrary to the authors' previous experiences, it was not possible, at higher values of

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n, to obtain the anticipated classical-type modes for the conical shells, although very careful slow-frequency sweeps were made with all forementioned shaker configurations for each mode investigated.

The methods of Goldberg were used to compute the response of one of the authors' free-free conical shells to a concentrated, simple harmonic forcing function placed normal to the shell at the major diameter. When the forcing frequency was only 5% higher than the natural frequency of the $n = 5$ mode, the nodal patterns produced were similar to those reported in the note. A similar pattern was also produced when the forcing frequency was 20% higher than the natural frequency for $n = 5$. A point of interest in this connection is the fact that a smooth curve can be drawn through the data points which depict the frequency-n relationship for the response at the major diameter in the authors' note. This was true for all four of the frustums tested and would seem highly unlikely if the responses measured were primarily forced responses.

It is felt that certain system parameters may prove to be critical in the case of free-free conical frustum shells. Among these are conicity, modal frequency density, material and acoustic damping, length-to-thickness ratio, and radius-to-thickness ratio. Although it may be possible to design an experiment that yields the anticipated classical mode shapes, variations of these parameters which might explain the existence of nodal patterns such as obtained during this investigation should be considered. For instance, in the case of Hu's experiments the modal frequency density was about one-half that of the authors' experiment. Closer proximity of adjacent natural frequencies in combination with material or acoustic damping may result in the inherent coupling of classical type modes to give the nodal patterns the authors obtained.

References

[1] Herr, R. W., "A wide-frequency-range air jet shaker," NACA TN 4060 (1957).

[2] Goldberg, J. E., Bogdanoff, J. L., and Alspaugh, D. W., "On the calculation of the modes and frequencies of vibration of pressurized conical shells," *AIAA 5th Annual Structural and Materials Conference* (AIAA, New York, 1964), pp. 243–249.

[3] Mixson, J. S. and Herr, R. W., "An investigation of the vibration characteristics of pressurized thin-walled circular cylinders partly filled with liquid," NASA TR R-145 (1962).

Comments on "Laminar Flow in Plane Wakes of a Conducting Fluid in the Presence of a Transverse Magnetic Field"

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In a recent note by Gupta[1], the familiar wake equation is analyzed with respect to asymptotic solutions for large x, the latter being the distance behind the trailing edge of an obstacle. Using the notations of the referenced note, the equation is

$$Uu_x = \nu u_{yy} - (\sigma\mu^2H_0^2/\rho)u \tag{1}$$

where u is the velocity defect, σ is the electrical conductivity, and H_0 is the applied transverse magnetic field not necessarily a constant. The boundary conditions are $u(x, \infty) = 0$ and $u_y(x, 0) = 0$.

Since the equation is linear one may attempt a solution of the form

$$u = f(x)F(x, y) \tag{2}$$

where the property of self-similarity is embedded in F. Inserting (2) into (1) yields

$$U[f'F + fF_x] = \nu fF_{yy} - (\sigma\mu^2H_0^2/\rho)fF \tag{3}$$

Whence decoupling of the functions produces

$$f = \exp[-\int_{x_0}^{x} \frac{\sigma\mu^2H_0^2}{\rho}\frac{1}{U}dx] \tag{4}$$

and

$$UF_x = \nu F_{yy} \tag{5}$$

The solution of (5) using the notations in the note becomes

$$F = \sum_{n=0}^{\infty} A_n \frac{x}{x_0}^{-n-(1/2)} g_n(\xi) \tag{6}$$

The magnetic modification factor is expressed by (4), which shows $f \approx (x/x_0)^{-c}$ if $H_0 \sim x^{-1/2}$ and $f \sim \exp(-bx)$ if H_0 is a constant so that in the latter case similarity is still preserved in direct contradistinction with the contention in the reference.

Reference

[1] Gupta, A. S., "Laminar flow is plane wakes of a conducting fluid in the presence of a transverse magnetic field," AIAA J. 1, 2391–2392 (1963).

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Reply by Author to L. S. Han

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The author wishes to thank Han for pointing out the possibility of another similarity solution of the wake equation [Eq. (4) of my note] for the case of a uniform magnetic field in the form

$$u = \exp(-bx)\cdot F(x, y) \tag{1}$$

where F satisfies

$$UF_x = \nu F_{yy} \tag{2}$$

This solution that is derived by Han in a slightly different way can also be obtained from the set of Eqs. (7) of my note if, instead of taking $UCC'/\nu = 2$, we equate UCC'/ν to zero, in which case $C(x)$ reduces to a nonzero constant. [$C = 0$ being impossible by virtue of the Eq. (5) of my note.] In this case the second equation of the set (7) leads to $a(x) \sim \exp(-bx)$, and the third equation becomes valid for a uniform magnetic field.

Unfortunately, however, the preceding Eq. (2) does not admit of a solution satisfying the boundary conditions $F(x, \infty) = 0$ [corresponding to $u(x, \infty) = 0$] and $F_y(x, 0) = 0$ [corresponding to $u_y(x, 0) = 0$]. This can be seen in the following way.

Let the solution of the foregoing Eq. (2) be taken in the form

$$F = F(\eta) \qquad \eta = yD(x) \tag{3}$$

so that Eq. (2) becomes

$$\eta F'(\eta) \times D'(x)/D(x) = \nu F''(\eta)/U \times D^2 \tag{4}$$

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